Emission of Back-to-Back Nucleons in Lepton-Nucleus Scattering

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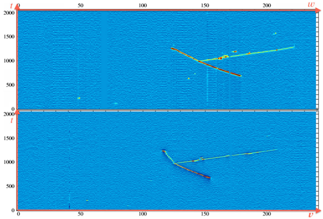
Outline

- ⋆ Motivation
- ⋆ Defining correlations in interacting many-body systems
- ⋆ Are nucleon-nucleon correlations observable?
- ⋆ Implications for the analysis of future experiments
- ⋆ Summary & prospects

Motivation

★ The recent observation of two-nucleon knock out events in the ArgoNeut detector [R. Acciarri *et al*, Phys. Rev. D **90**, 012008 (2014)] has revived the interest for short-range correlations in the nuclear ground state

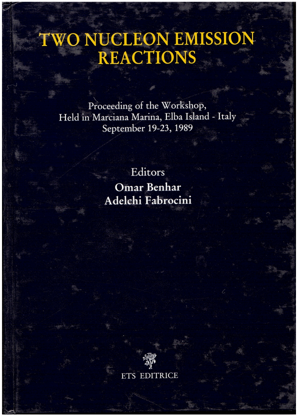
- 30 charged-current events with a pair of protons emitted at the interaction vertex
- 19 events with both protons carrying momenta larger the nuclear Fermi momentum (∼ 250 MeV)
- 4 events with back-to-back configuration *observed* in the final state



- 4 events with *reconstructed* back-to-back configuration of a proton-neutron pair in the initial state

Correlations & two-nucleon emission

- ⋆ Nucleon-nucleon correlations are a long standing and elusive issue. Their study through measurements of two-nucleon knock out processes has been extensively discussed since the dawning of the age of continuous electron beam accelerators

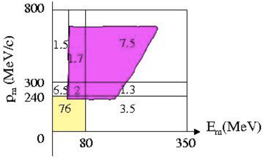


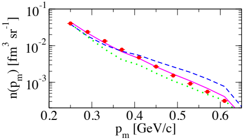
2p2h final states in neutrino-nucleus interactions

★ Slide extracted from a talk delivered at NUFACT11, University of Geneva, August 2011

Measured correlation strength

- the correlation strength in the 2p2h sector has been measured by the JLAB E97-006 Collaboration using a carbon target
- strong energy-momentum correlation: $E \sim E_{thr} + \frac{A-2}{A-1} \frac{\mathbf{k}^2}{2m}$





- Measured correlation strength 0.61 ± 0.06, to be compared with the theoretical predictions 0.64 (CBF) and 0.56 (G-Matrix)
- Correlated nucleons (most of the times a proton and a neutron) have momenta > 250 MeV, pointing in opposite directions

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Defining correlations

- Consider a system of N interacting particle described by the wave function $\Psi(x_1, \ldots, x_N)$, with $x_i \equiv (\mathbf{r}_i, \sigma_i)$
- Probability of finding particles $1, \ldots, n$ at positions $\mathbf{r}_1, \ldots, \mathbf{r}_n$

$$\rho^{(n)}(\mathbf{r}_1, \ldots, \mathbf{r}_n) = \frac{N!}{(N-n)!} \sum_{\sigma_1,\ldots,\sigma_N} \int d\mathbf{r}_{n+1} \ldots d\mathbf{r}_N \, |\Psi(x_1, \ldots, x_N)|^2$$

- Particles 1 and 2 are correlated if

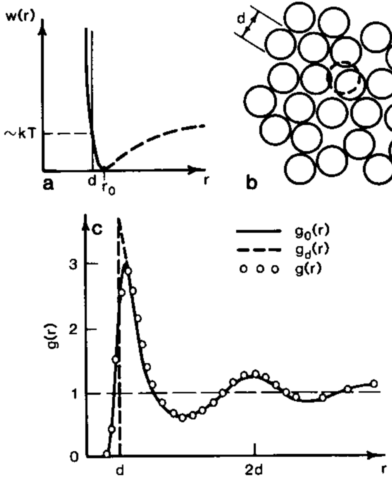
$$\rho^{(2)}(\mathbf{r}_1, \mathbf{r}_2) \neq \rho^{(1)}(\mathbf{r}_1)\rho^{(1)}(\mathbf{r}_2)$$

- The quantity

$$g(\mathbf{r}_1, \mathbf{r}_2) = \frac{\rho^{(2)}(\mathbf{r}_1, \mathbf{r}_2)}{\rho^{(1)}(\mathbf{r}_1)\rho^{(1)}(\mathbf{r}_2)}$$

provides a measure of correlations in coordinate space

The archetype corelated system: the Van der Waals liquid



- ⋆ Equation of state at particle density ρ and temperature T

$$P = \frac{\rho T}{1 - \rho b} - a\rho^2 ,$$

  - ▹ $b \propto d^3$ is the “excluded volume”
  - ▹ $a \sim$ integral of the attractive part of the interaction

- ⋆ The *full* Van der Waals potential provides a good description of atomic systems. Hovever, its use in perturbation theory involves insurmountable difficulties.

Coordinate vs momentum space

- ⋆ Bottom line: correlations are best defined in coordinate space.
- ⋆ To see this, consider a non interacting Fermi gas. The *joint* probability of finding two particles with momenta $\mathbf{k}_1$ and $\mathbf{k}_2$ is

$$n_{FG}(\mathbf{k}_1, \mathbf{k}_2) = \theta(k_F - |\mathbf{k}_1|)\theta(k_F - |\mathbf{k}_2|) \left[1 - \frac{1}{N} \frac{\rho}{2} \, (2\pi)^3 \delta(\mathbf{k}_1 - \mathbf{k}_2) \right]$$

- ⋆ *In the absence of long range order*, a similar result holds true in interacting systems

$$n(\mathbf{k}_1, \mathbf{k}_2) = n(\mathbf{k}_1) n(\mathbf{k}_2) \left[1 + O\left(1/N\right) \right]$$

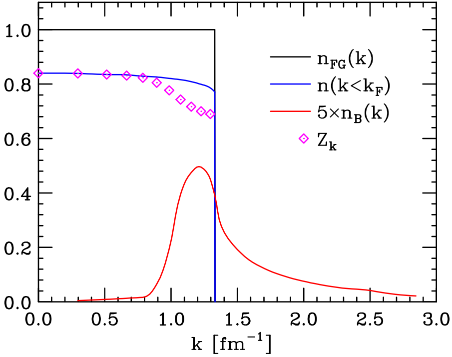
- ⋆ In momentum space, non trivial correlation effects on $n(\mathbf{k}_1, \mathbf{k}_2)$ vanish in the $N \to \infty$ limit. However, correlations strongly affect the behaviour of $n(\mathbf{k})$ at $|\mathbf{k}| > k_F$.

Momentum distribution of interacting Fermi systems

★ The momentum distribution can be split into quasi particle (pole) and and correlation (continuum) contributions in a *model independent* fashion

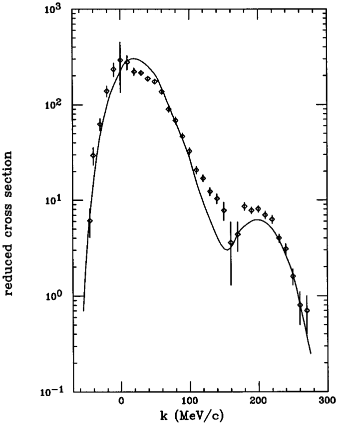
$$n(\mathbf{k}) = \int dE\, P(\mathbf{k}, E) = Z_k \theta(k_F - |\mathbf{k}|) + n_B(\mathbf{k})$$

★ Isospin-symmetric nuclear matter at equilibrium density, as an example



Mmentum distribution of shell model state

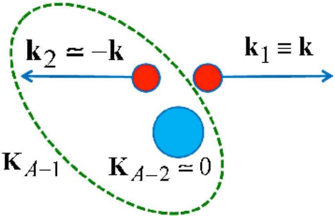
- Momentum distribution of the $3s$ state of ^{208}Pb , extracted from $(e, e'p)$ data

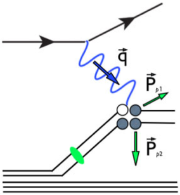


- The occurrence of high momentum nucleons is a distinctive evidence of correlation effects
- Back in 1952 AD, Blatt & Weiskopf warned their readers that
  - "The limitation of any independent particle model lies in its inability to encompass the correlation between the positions and spins of the various particles in the system"

Probing correlations through single nucleon knock out

- ⋆ Consider quasi elastic (QE) process $\ell + A \to \ell' + N + (A-1)^*$
- ⋆ Note that, on account of nucleon-nucleon correlations in the initial state, $(A-1)^*$ *is not necessarily a bound state*. It may consist of a bound $(A-2)$-nucleon system plus one nucleon excited to the continuum





- ⋆ The two emitted nucleons carry momenta $-\mathbf{k}$ and $\mathbf{k}+\mathbf{q}$, $\mathbf{q}$ being the momentum transfer
- ⋆ Establishing the occurrence of correlated pairs in the initial state requires the *reconstruction* of the initial momentum of the struck nucleon

Knock out of a correlated nucleon (continued)

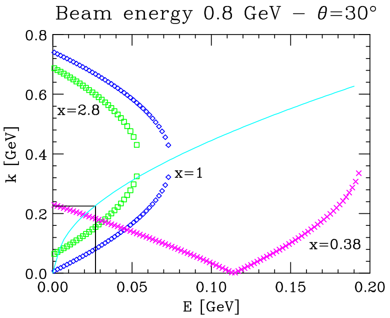
- ⋆ Within the impulse approximation (IA), the double-differential cross section in the QE channel can be written as

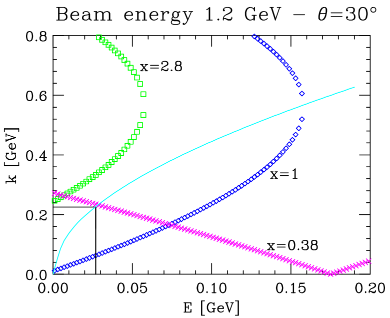
$$\frac{d\sigma_A}{d\Omega_{\mathbf{k}'}d\omega} \propto \frac{1}{q} \int_{E_{\rm min}}^{E_{\rm max}} dE \int_{k_{\rm min}}^{k_{\rm max}} kdk\; P(k,E) \frac{d\sigma_N}{d\Omega_{\mathbf{k}'}d\omega}$$

- ⋆ The spectral function $P(k,E)$ describes the energy and momentum distribution of the struck nucleon

$$n(k) = \int dE\; P(k,E)$$

- ⋆ Probing high momentum components, arising from short-range nucleon-nucleon correlations, requires $k_{\rm min} > k_F \sim 250$ MeV and $E_{\rm min} > \bar{\epsilon} \sim 25$ MeV





- $\sim 10\%$ of the inclusive strength in the two-nucleon emission sector
- $\sim 14\%$ of the inclusive strength in the two-nucleon emission sector

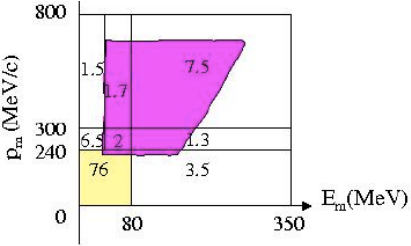
★ Recall

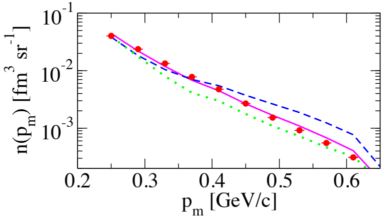
- $x \lessgtr 1 \leftrightharpoons \omega \gtrless \omega_{\rm QE} \approx Q^2/2m_N$
- $x \lessgtr 1 \leftrightharpoons \hat{\mathbf{k}} \cdot \hat{\mathbf{q}} \approx \pm 1$
- energy and momentum are strongly correlated through

$$E \approx E_{\rm thr} + \frac{A-2}{A-1}\,\frac{k^2}{2m_N}$$

Measured correlation strength

- ★ The correlation strength in the 2p2h sector has been measured by the JLAB E97-006 Collaboration using a carbon target





- ★ Measured correlation strength

Experiment	0.61 ±0.06
Greens function theory [3]	0.46
CBF theory [2]	0.64
SCGF theory [4]	0.61

Impact on neutrino energy reconstruction

- In the charged current quasi elastic (CCQE) channel, assuming single knock out, the *reconstructed* neutrino energy can be written as

$$E_\nu = \frac{m_p^2 - m_\mu^2 - E_n^2 + 2E_\mu E_n - 2\mathbf{k}_\mu \cdot \mathbf{p}_n + |\mathbf{p}_n^2|}{2(E_n - E_\mu + |\mathbf{k}_\mu| \cos\theta_\mu - |\mathbf{p}_n| \cos\theta_n)} ,$$

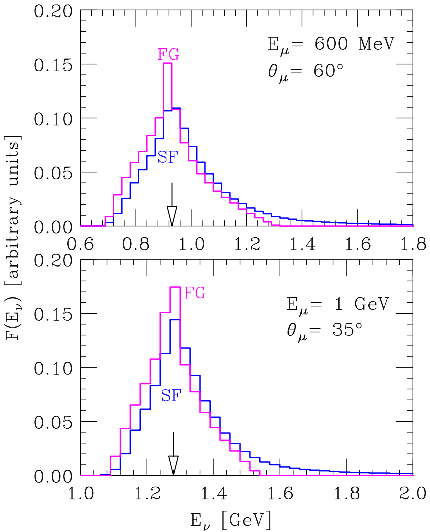
  where $|\mathbf{k}_\mu|$ and θ_μ are measured kinematical variables of the outgoing charged lepton, while $\mathbf{p}_n$ and E_n are the momentum and energy of the interacting neutron
- Energy conservation requires

$$E_n = M_A - E^*_{A-1} = M_A - \sqrt{(M_A - m_n + E)^2 + |\mathbf{p}_n|^2} \approx m_n - E$$

- In interactions involving a correlated nucleon, the excitation energy of the recoiling nucleus is large, typically $E \sim 50 - 100$ MeV
- The occurrence of these processes lead to the appearance of a high energy tail in the neutrino energy distribution

Reconstructed neutrino energy in the CCQE channel

- ⋆ Neutrino energy reconstructed using 2×10^4 pairs of $(|\mathbf{p}_n|, E_n)$ values sampled from realistic (SF) and FG oxygen spectral functions
- ⋆ The average value $\langle E_\nu \rangle$ obtained from the realistic spectral function turns out to be shifted towards larger energy by $\sim$ 70 MeV



Spin-isospin dependence of correlations

- ⋆ Correlations reflect the complex structure of the nucleon-nucleon force, which is known to be strongly spin-isospin dependent & non spherically symmetric
- ⋆ Correlations are strongest in proton-neutron pairs with total spin and isospin $S = 1$ and $T = 0$. In these *deuteron-like* pairs—also called *quasi deuterons* (QD)—non central interactions play an important role
- ⋆ Experimental information on QD pairs has been extracted from the measured nuclear photoabsorption cross section, written in the form

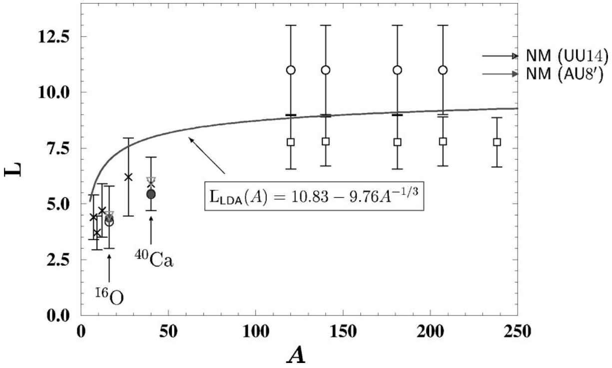
$$\sigma_A(E_\gamma) = \mathcal{P}_D \, \sigma_{QD}(E_\gamma)$$

where E_γ is the photon energy and $\mathcal{P}_D$ is interpreted as the effective number of QD pairs

$$\mathcal{P}_D = \mathrm{L} \left[\frac{Z(A-Z)}{A} \right]$$

Quasi deuteron pairs in nuclei

- ⋆ A-dependence of the Levinger's factor, L , extracted from the data, compared to results of *ab initio* many-body calculations



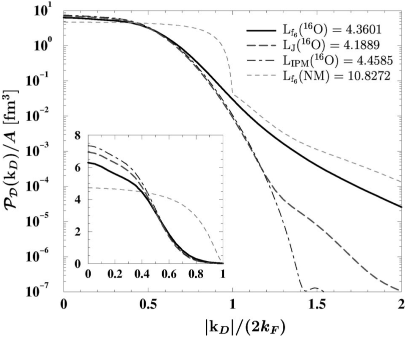
- ⋆ The theoretical predictions for oxygen and calcium turn out to be in good agreement with the data

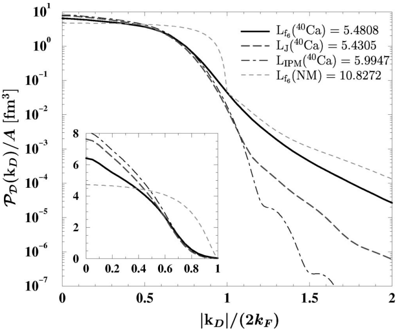
Quasi deuteron distributions and Levinger's factor

- ⋆ Theoretical predictions of the Levinger's factors are obtained from

$$L = \frac{1}{Z(A-Z)} \mathcal{P}_D = \frac{3}{Z(A-Z)} \int \frac{d^3k_D}{(2\pi)^3} \mathcal{P}_D(\mathbf{k}_D)$$

where $\mathcal{P}_D(\mathbf{k}_D)$ provides the probability of finding a QD pair with center-of-mass momentum $\mathbf{k}_D$



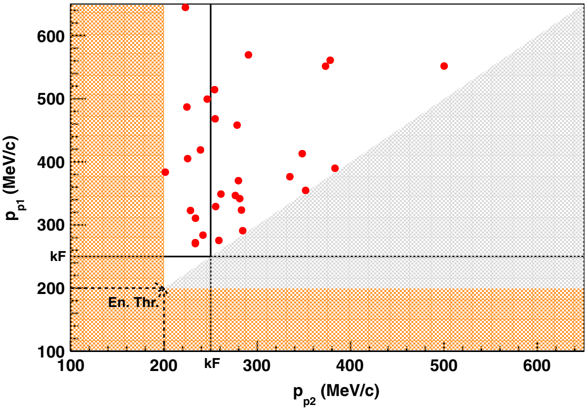


Summary & Prospects

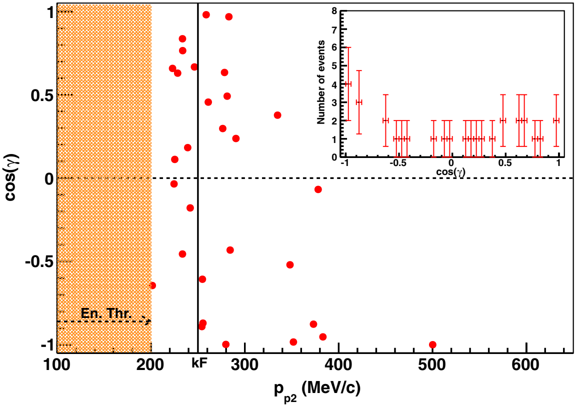
- ⋆ The observation of events with emission of two-nucleons carrying momenta exceeding the Fermi momentum provides new compelling evidence of strong nucleon-nucleon correlations.
- ⋆ The existing studies of the momentum distribution of QD pairs in nuclei may provide some guidance for the interpretation of the events in which the reconstructed initial configuration does not correspond to a back-to-back correlated pair.
- ⋆ Mechanisms other than knock out of a correlated nucleon need to be analysed within a consistent framework
- ⋆ Correlation effects must be included in any nuclear model aimed at describing neutrino-nucleus interactions
- ⋆ The observation of two nucleon emission events has clearly shown the potential of the Liquid Argon Time Projection Chamber technology. The upcoming measurement of the argon spectral function at Jefferson Lab will provide new information, needed to fully exploit this potential.

Backup slides

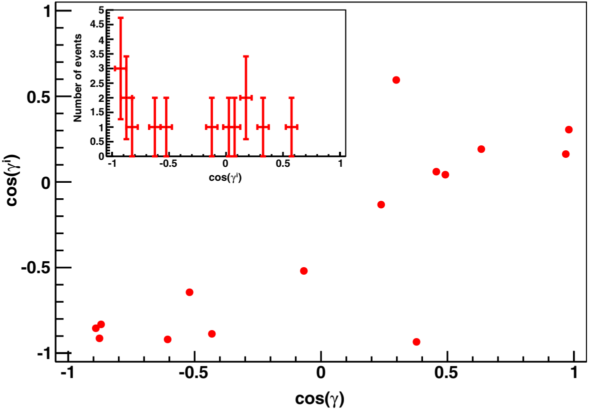
ArgoNeut events



ArgoNeut events



ArgoNeut events



Model independent determination of correlations

- Definition of Green's function

$$iG(x-x') = \langle 0|T[\hat{\psi}(x)\hat{\psi}^\dagger(x')]|0\rangle$$

After Fourier transformation ($\eta = 0^+$)

$$G(\mathbf{k},E) = \sum_n \left\{ \frac{|\langle n_{(N+1)}(\mathbf{k})|a^\dagger_{\mathbf{k}}|0_N\rangle|^2}{E-(E_n-E_0)+i\eta} + \frac{|\langle n_{(N-1)}(-\mathbf{k})|a_{\mathbf{k}}|0_N\rangle|^2}{E+(E_n-E_0)-i\eta} \right\}$$

$$= G_p(\mathbf{k},E) + G_h(\mathbf{k},E) = \int dE' \left[\frac{P_p(\mathbf{k},E')}{E-E'+i\eta} + \frac{P_h(\mathbf{k},E')}{E+E'-i\eta} \right]$$

- Spectral functions of hole and particle states

$$P_h(\mathbf{k},E) = \sum_n |\langle n_{(N-1)}(\mathbf{k})|a_{\mathbf{k}}|0_N\rangle|^2 \delta(E-E_n+E_0) = \frac{1}{\pi}\,\mathrm{Im}\,G_h(\mathbf{k},E)$$

$$P_p(\mathbf{k},E) = \sum_n |\langle n_{(N+1)}(\mathbf{k})|a^\dagger_{\mathbf{k}}|0_N\rangle|^2 \delta(E+E_n-E_0) = \frac{1}{\pi}\,\mathrm{Im}\,G_p(\mathbf{k},E)$$

Analytic structure of the Green's function

- ⋆ In interacting systems, the Green's function (e.g. for hole states) can be written in terms of the particle self energy $\Sigma(\mathbf{k}, E)$

$$G_h(\mathbf{k}, E) = \frac{1}{E - |\mathbf{k}|^2/2m - \Sigma(\mathbf{k}, E)}$$

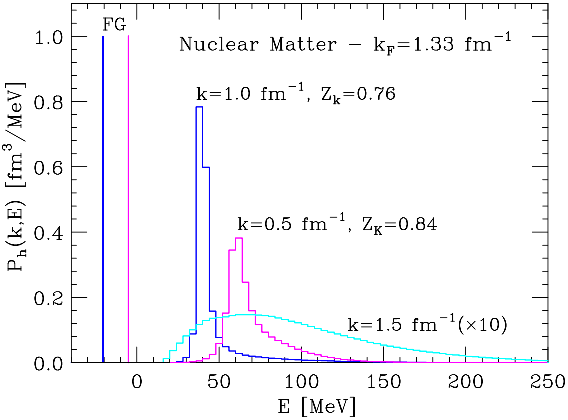
- ⋆ Landau's quasiparticle picture: isolate contributions of $1h$ (*bound*) intermediate states, exhibiting poles at energies ϵ_k, given by $\epsilon_k = |\mathbf{k}|^2/2m + \text{Re}\,\Sigma(\mathbf{k}, \epsilon_k)$, as $\text{Im}\,\Sigma(\mathbf{k}, E) \to 0$ (Fermi surface)
- ⋆ The resulting expression is

$$G_h(\mathbf{k}, E) = \frac{Z_k}{E - \epsilon_k - iZ_k\,\text{Im}\,\Sigma(\mathbf{k}, e_k)} + G_h^B(\mathbf{k}, E)$$

where $Z_k = |\langle -\mathbf{k}|a_{\mathbf{k}}|0\rangle|^2$, and $G_h^B(\mathbf{k}, E)$ is a smooth contribution, arising from $2h - 1p, 3h - 2p, \ldots$ (*continuum*) intermediate states

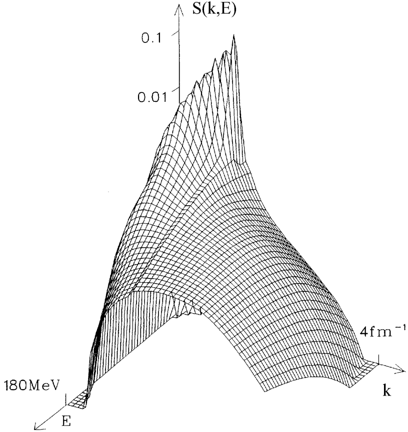
Hole spectral function of nuclear matter from CBF

$$P_h(\mathbf{k}, E) = \frac{1}{\pi} \frac{Z_k^2 \ \mathrm{Im}\ \Sigma(\mathbf{k}, \epsilon_k)}{[E - \mathbf{k}^2/2m - \mathrm{Re}\ \Sigma(\mathbf{k}, E)]^2 + [Z_k \mathrm{Im}\ \Sigma(\mathbf{k}, \epsilon_k)]^2} + P_h^B(\mathbf{k}, E)$$



Spectral function of infinite nuclear matter

★ Results obtained using CBF perturbation theory and the U14+TNI hamiltonian



★ The correlation contribution can be identified by its distinctive energy dependence

Exploiting the (near) universality of correlations

- Local density approximation

$$P(\mathbf{k},E) = P_{MF}(\mathbf{k},E) + P_{corr}(\mathbf{k},E)$$

  - $P_{MF}(\mathbf{k},E) \rightarrow$ from $(e,e'p)$ data

$$P_{MF}(\mathbf{k},E) = \sum_n Z_n |\phi_n(\mathbf{k})|^2 \, F_n(E - E_n)$$

  - $P_{corr}(\mathbf{p},E) \rightarrow$ from uniform nuclear matter calculations at different densities:

$$P_{corr}(\mathbf{k},E) = \int d^3r \, \rho_A(r) \, P^{NM}_{corr}(\mathbf{k},E;\rho = \rho_A(r))$$

- Widely and successfully employed to analize (e,e') data at beam energies $\sim 1GeV$
- Warnings: model dependence, chance of double counting